

Reference: F&H chapter 18

Introduction:

- Over half the known nuclei have configurations **(Z,N) even, $J^\pi = 0^+$**
- Recall that an empirical **pairing term** is included the semi-empirical mass formula to account for their **unusual stability**.

N.B. The pairing term is **not** accounted for in the shell model, which ignores all interactions between particles!!!
- It costs too much energy to break a pair of nucleons and populate higher single particle states, so the excitations of even-even nuclei tend to be of a **collective nature**
 - the nuclear matter distribution as a whole exhibits quantized vibrations in some cases and rotations in others, with characteristic frequency patterns.
- Vibrational spectra are seen in nuclei that have an intrinsic **spherical shape**
- Rotational excitations tend to occur in nuclei with permanent **quadrupole deformations**

Model: quantized oscillations of a liquid droplet at constant density
 (why? repulsive short-distance behaviour of the N-N force!)

Consider oscillations about a spherical equilibrium shape, with a time-dependent boundary surface expressed as a linear combination of spherical harmonic functions:

$$R(\theta, \phi, t) = R_o \left\{ 1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}(t) Y_{\lambda\mu}(\theta, \phi) \right\}$$



expansion describes any shape at all, given appropriate coefficients. Each contribution can in principle oscillate at a different frequency....

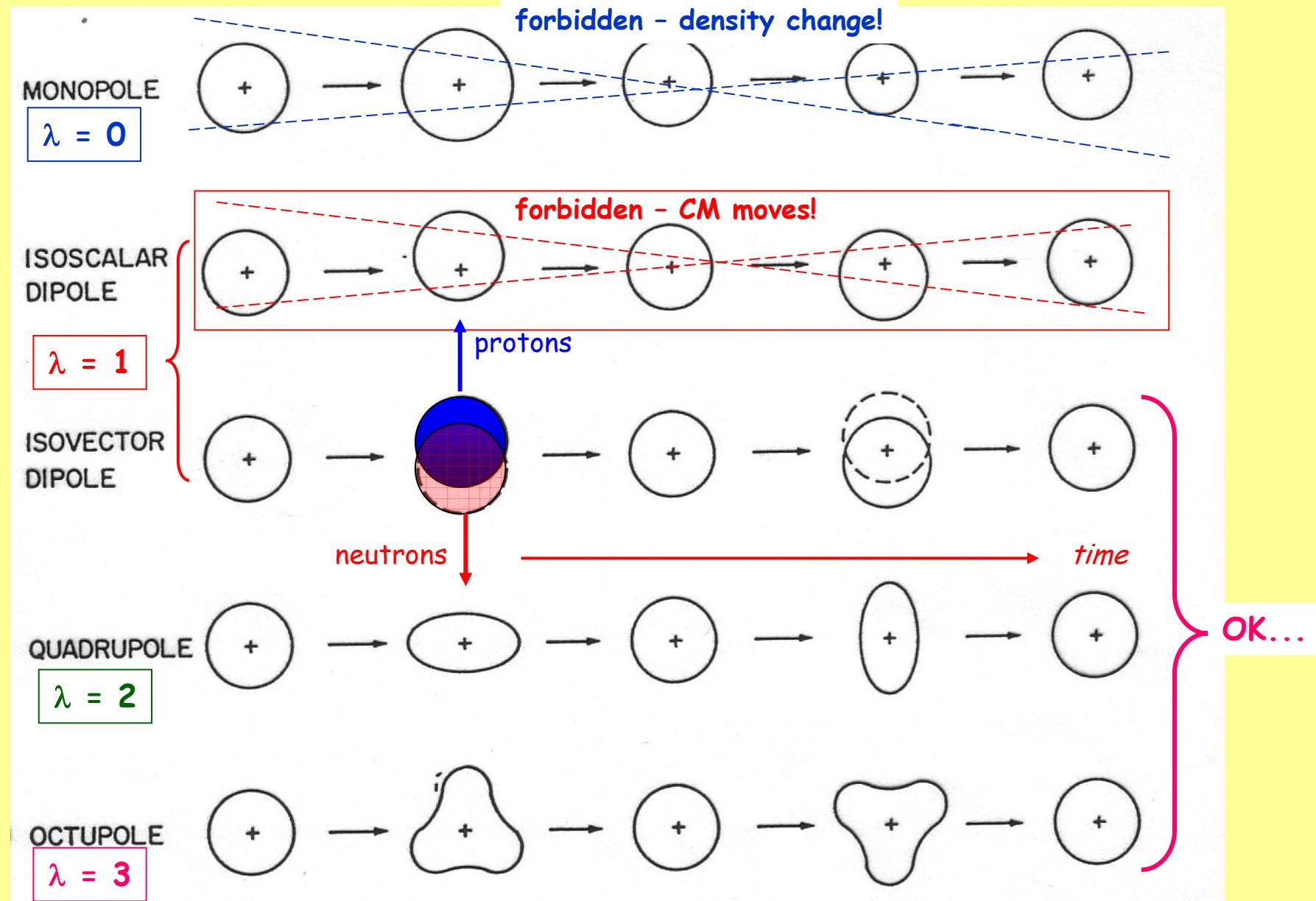
Normal modes of the system correspond to excitations with a particular value of λ and μ , and these will occur at characteristic frequencies.

Application to nuclei:

1. restriction to axial symmetry, i.e. $\mu = 0$
2. vibrations are quantized: $E_n = n \hbar\omega$

Illustration: time sequence of oscillating nuclear shapes

3



- characteristic feature that can be seen in all nuclei
- very short-lived state at high excitation
- $E_1 = \hbar\omega_1 \approx 78 A^{-1/3}$ MeV (example below: ^{197}Au , $E = 15$ MeV)
- $\Gamma \approx 6$ MeV (common feature) $\rightarrow \tau \approx 10^{-22}$ s
- classical analog is an oscillating electric dipole moment - can decay via E1 radiation pattern (signature)

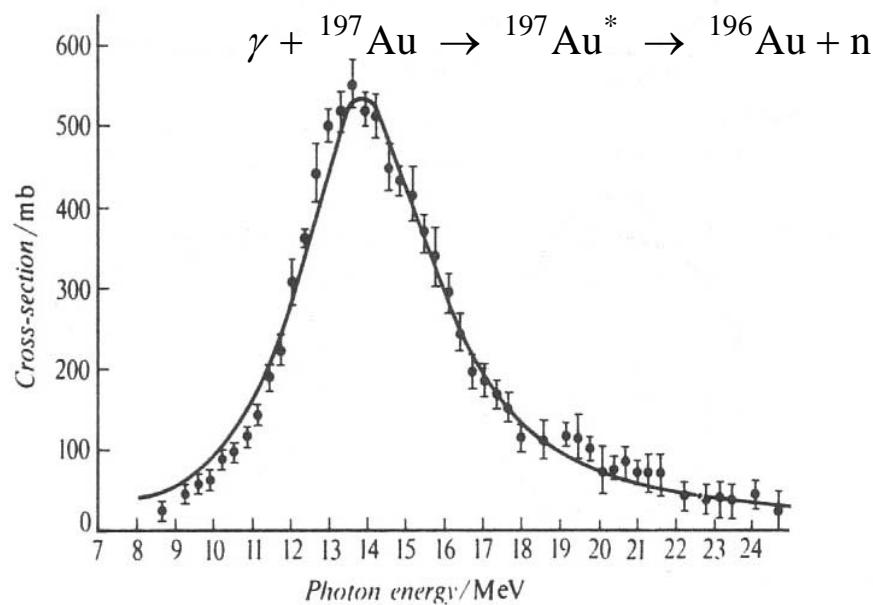


Fig. 8.10 Giant resonance of photodisintegration in ^{197}Au . The yield of neutrons is shown as a function of the energy of the monochromatic photons used to produce the reaction (Fultz, S. C. et al., Phys. Rev., **127**, 1273, 1963).

Electric dipole radiation pattern:

Griffiths, Intro. to Electrodynamics:

$$\langle \vec{S} \rangle \sim \frac{\sin^2 \theta}{r^2} \hat{r}$$

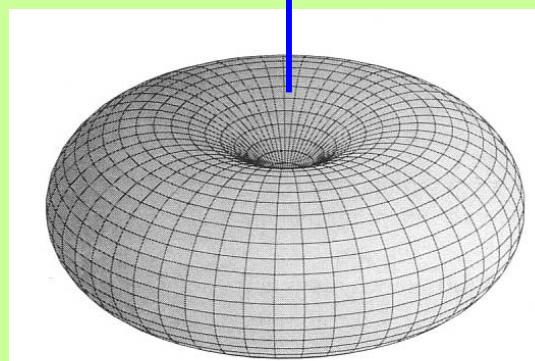
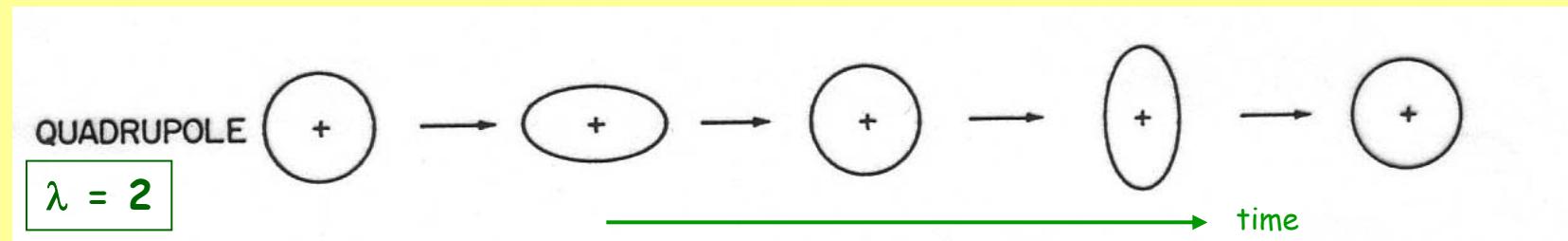
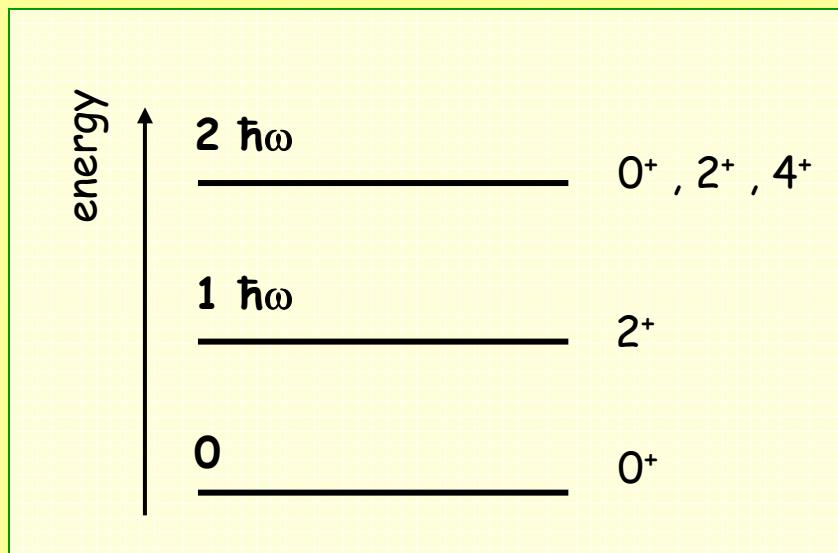


Figure 11.4



- typically, $\hbar\omega_2 \sim 1$ MeV in a variety of even-even nuclei
- excitation energy is low, so can expect to see up to several "quadrupole phonons" in the spectrum
- Boson excitations, so require a symmetric wave function under exchange of "particle" (phonon) labels → this restricts the total J^π ,



e.g. for two phonons:

$$(\vec{J} + \vec{J})_{symmetric} = \vec{J}$$

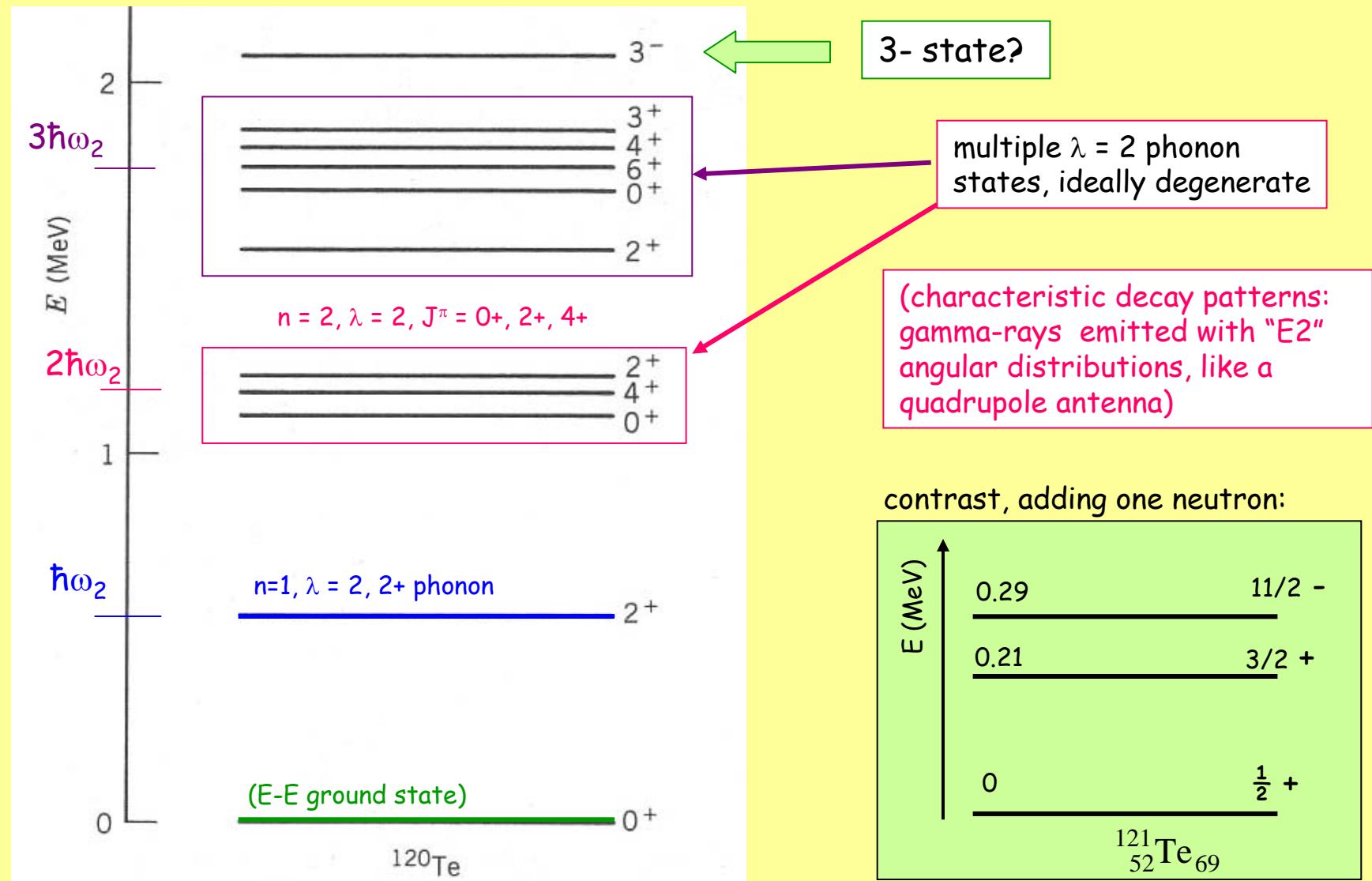
$$\Rightarrow J = 0, 2, 4 \text{ only}$$

← Model spectrum

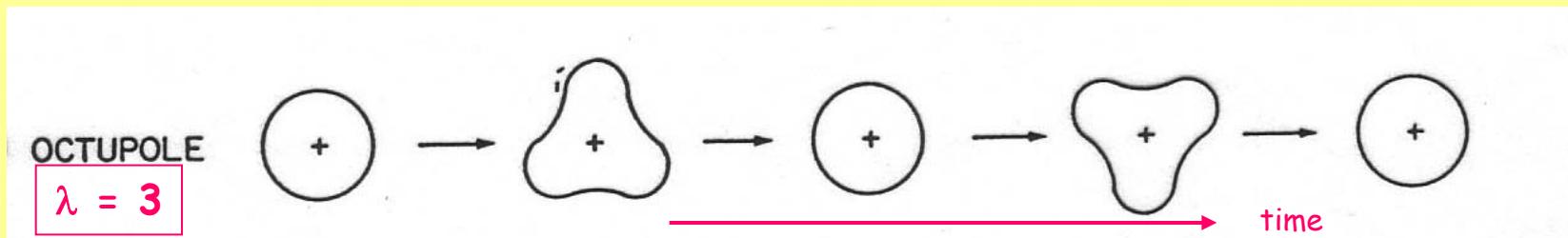
Example of vibrational excitations:

$^{120}_{52}\text{Te}_{68}$

6



The 3^- state is an octupole phonon, $\lambda = 3$:



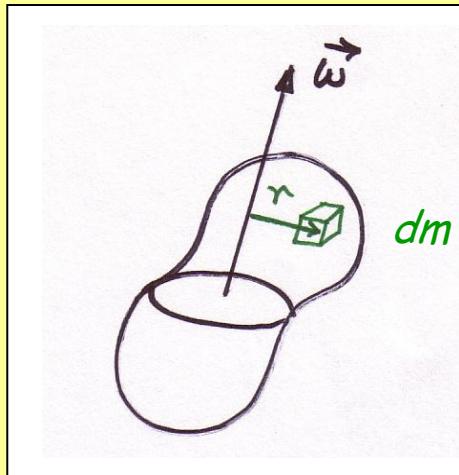
- $J^\pi = 3^-$
- $\hbar\omega_3 \sim (2-3) \hbar\omega_2 \sim 2 - 3 \text{ MeV}$
- typically only see one octupole phonon per spectrum

Summary:

- low lying excitations in even-even spherical nuclei have the same characteristic pattern up to a few MeV in excitation energy:
0+ (gs), 2+ (quadrupole phonon), (0+, 2+, 4+) (two phonons), 3- (octupole)

2. Quantized Rotations in deformed nuclei:

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Classical rotating object:

$$E = \frac{L^2}{2I}$$

$$I = \int r^2 dm$$

Replace L with **rotational angular momentum J**:

moment of inertia
sets the scale of the
energy level pattern

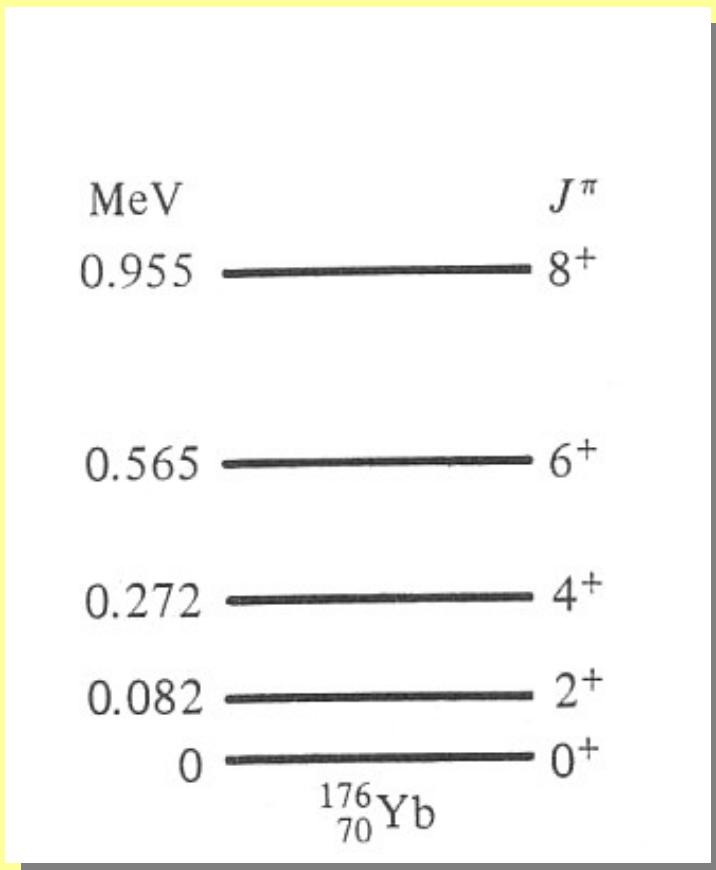
$$|J^2| = J(J+1) \Rightarrow E_J = \frac{\hbar^2}{2I} J(J+1)$$

allowed J determine
a characteristic
spacing pattern

J is quantized; "**rotational bands**" are spectra characterized by a given value of the **moment of inertia, I**, and a series of energy levels with $\Delta J = 1$ or 2 :

- even-even nucleus: $J = (0, 2, 4, 6, 8, 10, \dots)$ $\pi = +$
- odd-even deformed nucleus: $J = \frac{1}{2}$ integer, $\Delta J = 1$ within a "band"

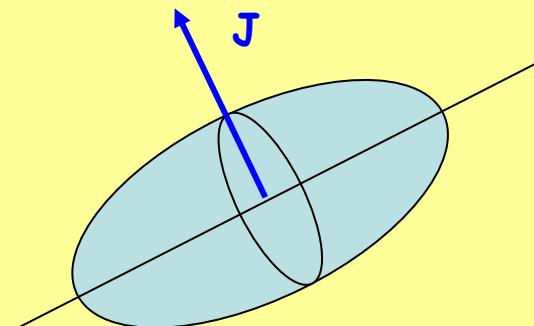
Quantized energy states of a rotating football!



Note -- larger I means smaller energy level spacing!

$$E_J = \frac{\hbar^2}{2I} J(J+1)$$

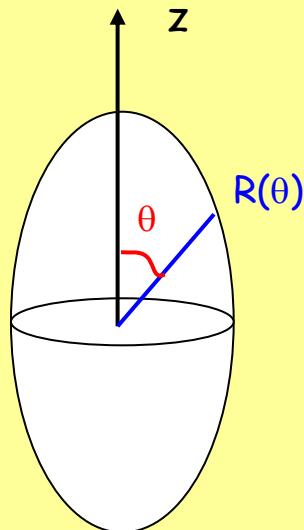
$$\Rightarrow \frac{\hbar^2}{2I} \approx 0.014 \text{ MeV}$$



note: rotations around the symmetry axis are indistinguishable; rotational angular momentum must be perpendicular to the symmetry axis.

The moment of inertia gives a measure of the nuclear shape:

10



parameterize the shape, quadrupole moment and moment of inertia assuming constant density football shape:

$$R(\theta) = R_o (1 + \beta Y_{20}(\theta))$$

$$\beta = \frac{4}{3} \sqrt{\frac{\pi}{5}} \frac{\Delta R}{R_o} \approx 1.05 \frac{\Delta R}{R_o}$$

$$Q_{zz} = \frac{3}{\sqrt{5\pi}} R_o^2 Z \beta (1 + 0.16\beta)$$

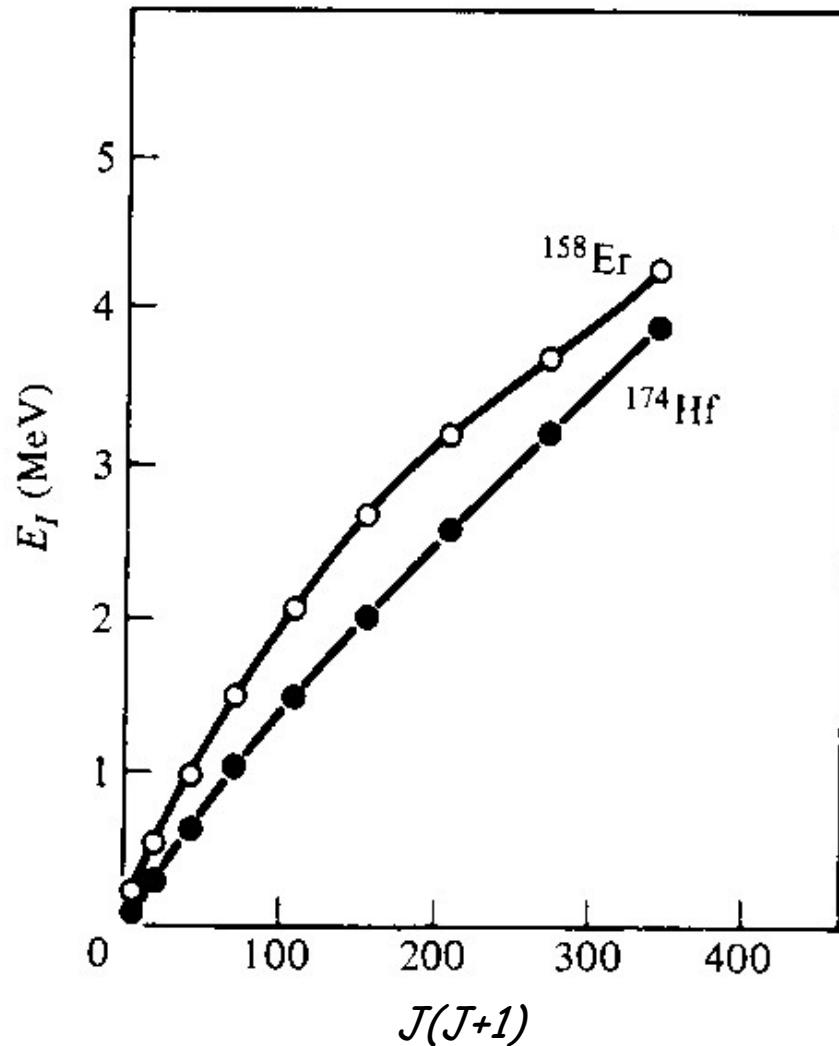
If the nucleus rotates like a solid: (*rigid body model*):

$$I_R = \frac{2}{5} M R_o^2 (1 + 0.31\beta)$$

If the nucleus rotates like a liquid drop: (*rotating fluid model*)

$$I_F = \frac{9}{8\pi} M R_o^2 \beta$$

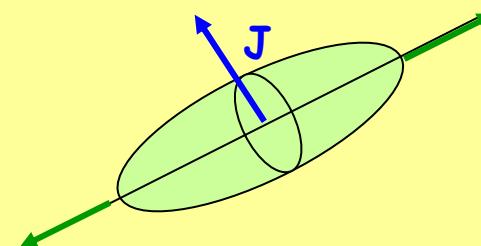
reality is somewhere in between...



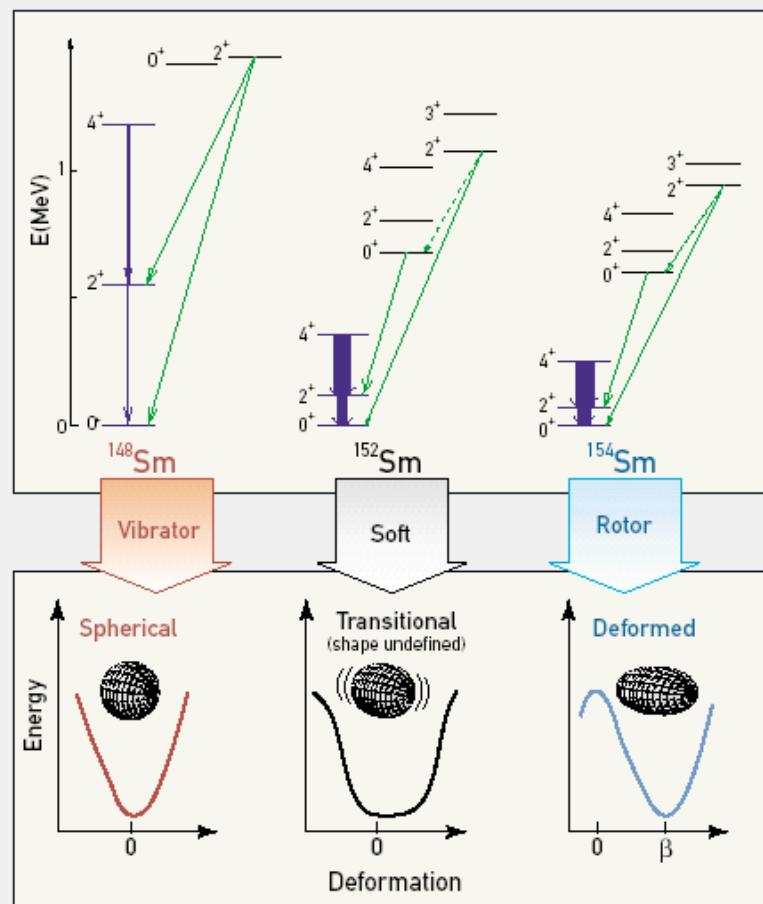
spectral analysis:

a plot of E vs $J(J+1)$ should give a straight line with slope $\hbar^2/2I$

- confirmed for ^{174}Hf
- but for ^{158}Er , the slope decreases (moment of inertia increases) with increasing J ...

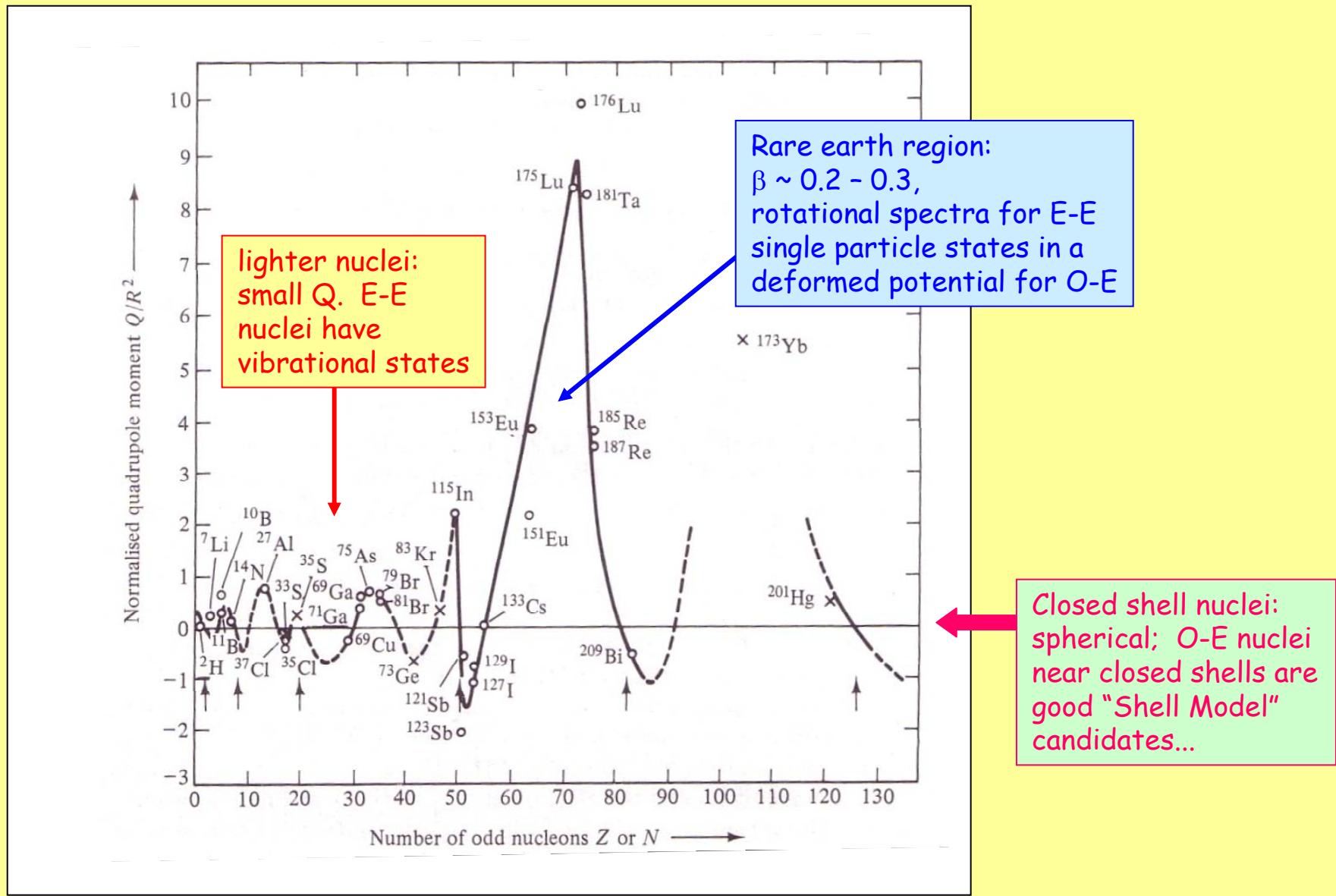


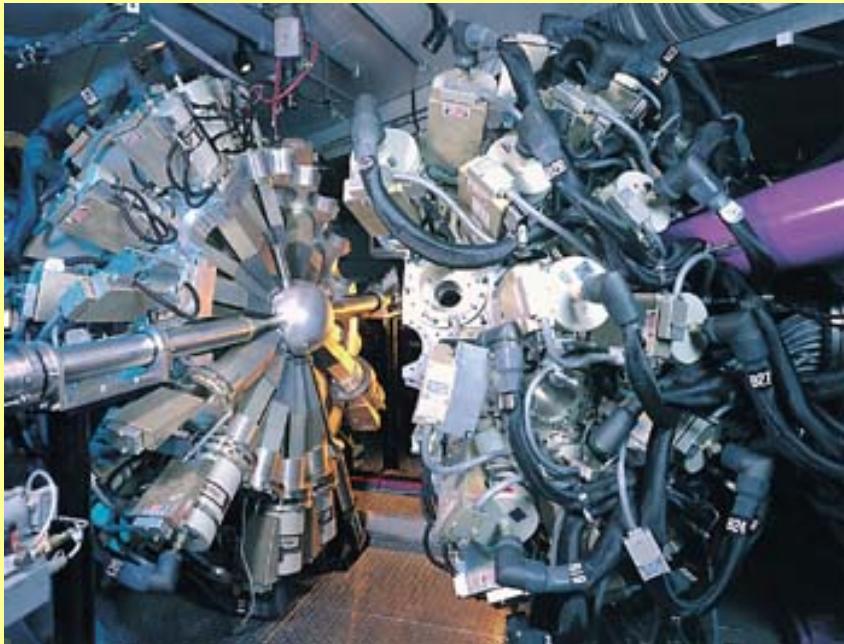
Like a rotating fluid:
"centrifugal stretching" along
the symmetry axis occurs for
larger angular momentum!



[web link: see lecture 1](#)

Shapes of samarium. The energy-level schemes for three excited isotopes of samarium are shown at the top of the figure, and the inferred nuclear shapes are shown below. The isotope ^{148}Sm shows features characteristic of a spherical vibrator, whereas ^{154}Sm exhibits rotational bands typical of a deformed (elongated) nucleus. On the other hand, ^{152}Sm behaves like a critical-point system, whose shape cannot be precisely defined. This is illustrated in the energy diagrams below the shapes, where energy is plotted as a function of shape deformation. Well-defined minima exist for two of the isotopes, but the energy minimum for ^{152}Sm is very broad, and it is impossible to say whether the nucleus is spherical or deformed.





Gammasphere Detector

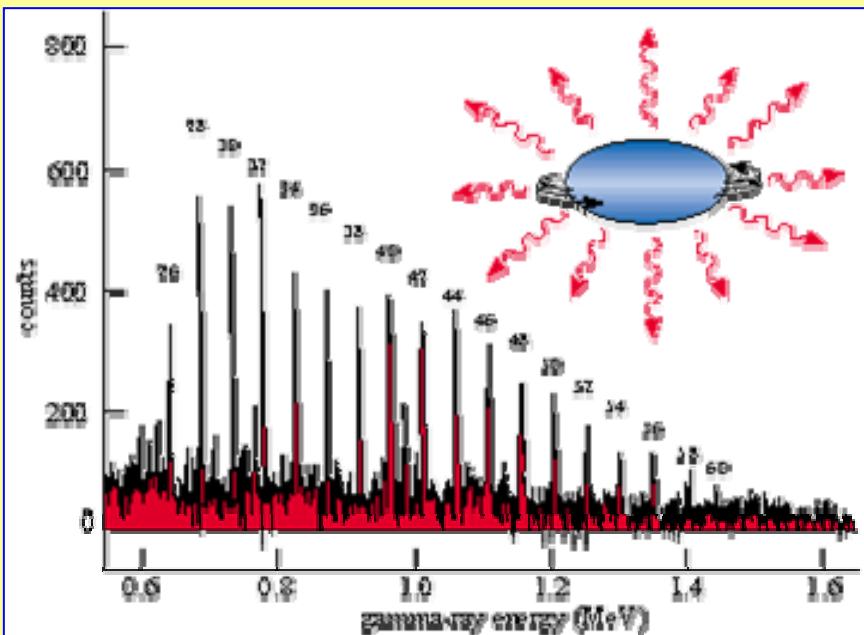
e.g. "8 π " detector array at TRIUMF,
"EUROBALL" array in France, "Gammasphere" UK

- high resolution detectors: Ge crystals
- surrounded by low resolution, high efficiency scintillation detectors (typically NaI) to reject events where only part of the energy is deposited in the Ge crystal due to Compton scattering
- full angular coverage for both \rightarrow "8 π detection!"

Great current interest:
"Superdeformed Nuclei" (2:1 axis ratio)

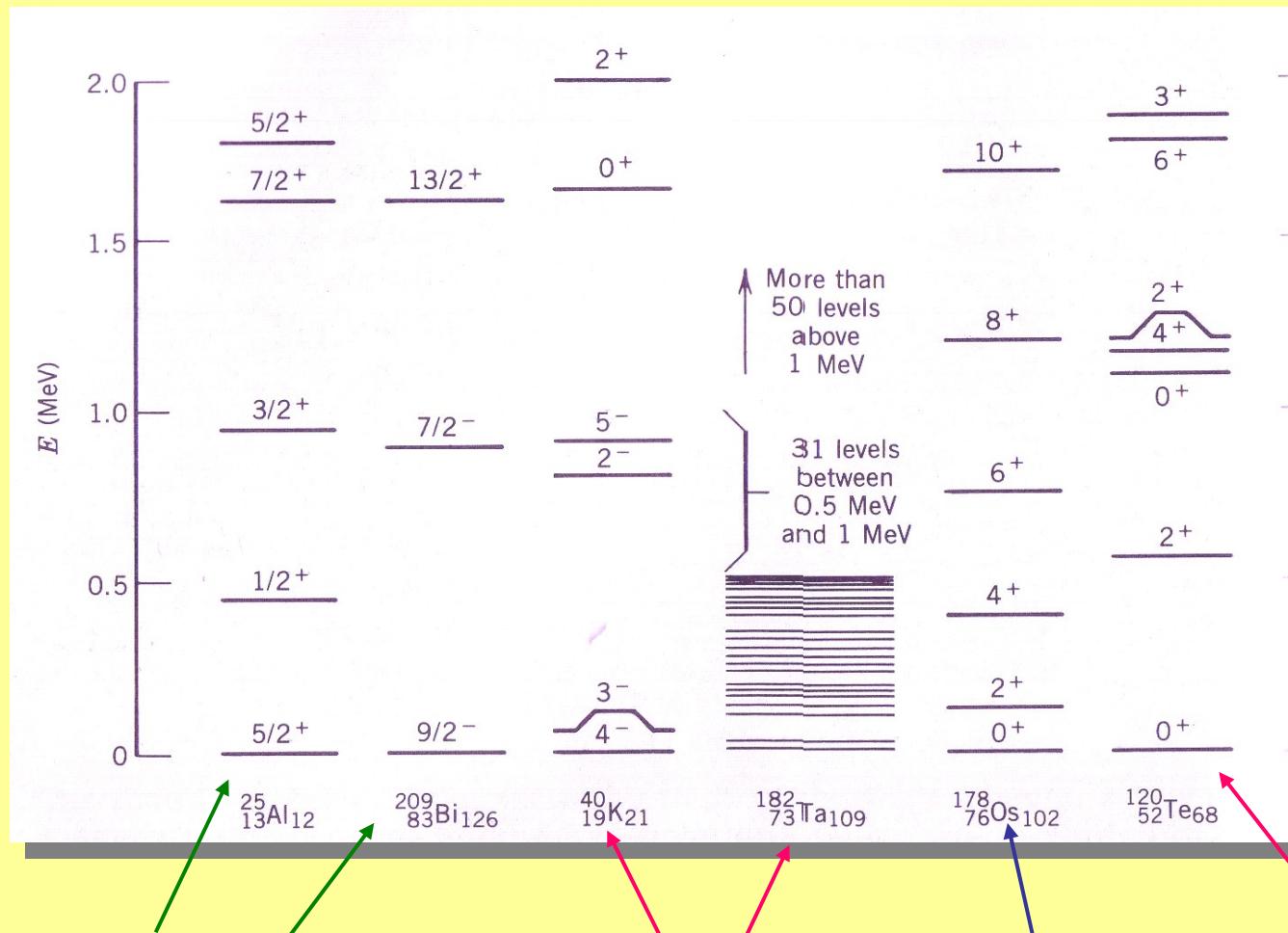
Physics World, July 1998: Superdeformed nuclei of dysprosium-152 decay by emitting a regular spectrum of gamma-rays. The number above each transition is the angular momentum quantum number, which decreases by two each time a photon is emitted. The photon carries h/p of angular momentum away from the nucleus, which slows down the rotation. After emitting approximately 20 such gamma-rays the nucleus abruptly loses its deformation.

Data from CLRC Daresbury Laboratory, UK



Example - nuclear states:

15



light nuclei, 1 valence nucleon
- good shell model description

odd-odd - more than one valence nucleon -- complicated!!!

rotational spectrum

vibrational spectrum

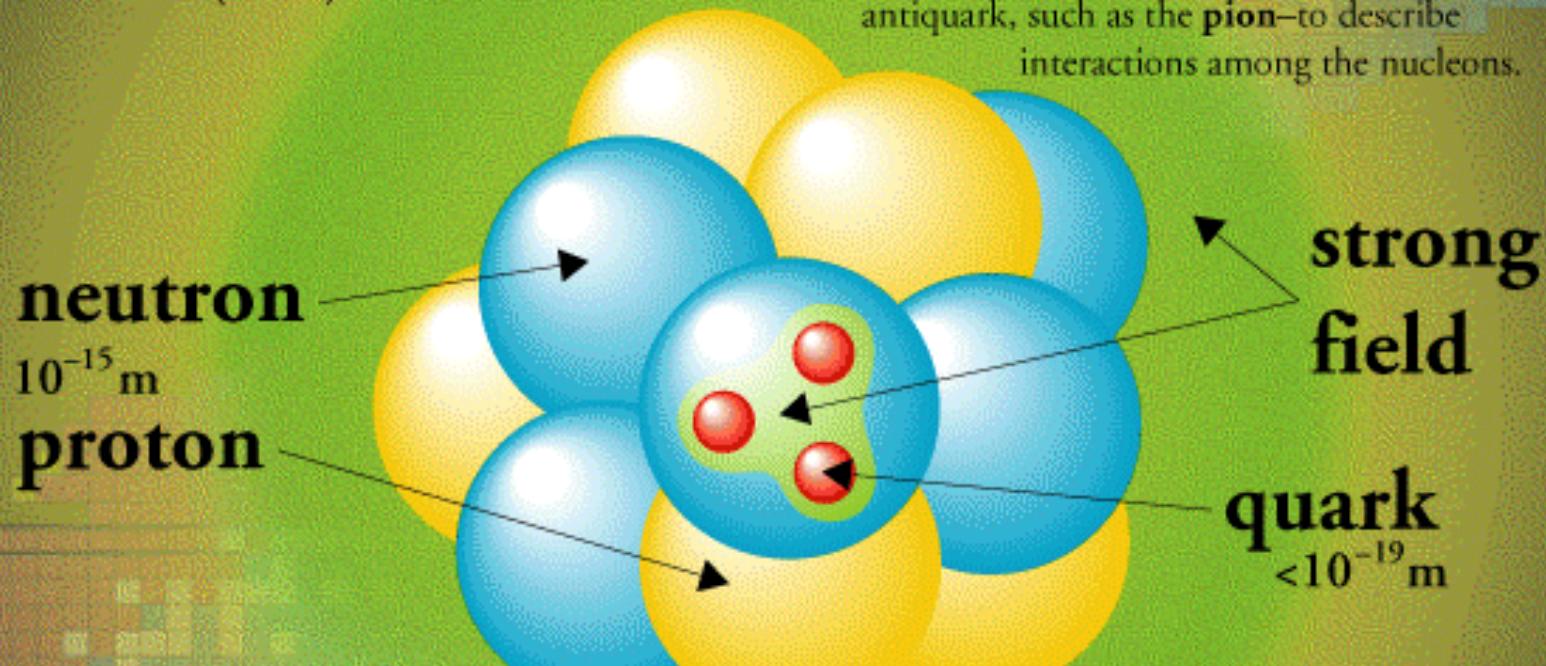
16.451:

The end of the Introduction!

The Nucleus

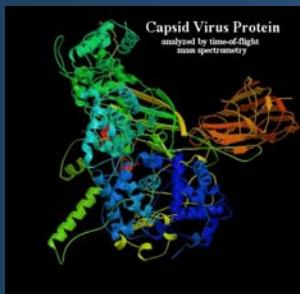
$(1\text{-}10) \times 10^{-15}\text{ m}$

At the center of the atom is a nucleus formed from **nucleons**—protons and neutrons. Each nucleon is made from three **quarks** held together by their strong interactions, which are mediated by gluons. In turn, the nucleus is held together by the **strong** interactions between the gluon and quark constituents of neighboring nucleons. Nuclear physicists often use the exchange of mesons—particles which consist of a quark and an antiquark, such as the **pion**—to describe interactions among the nucleons.



From the U.S. Long range plan : “Opportunities in Nuclear Science” (lecture 1):

The nucleus is a remarkable quantal system displaying diverse and rare phenomena. Governed by the strong interactions among nucleons, nuclei exhibit correlations resulting in both single-particle and collective modes of excitation. Nuclear structure theory attempts to understand these excitations and the responses of nuclei to different external probes, within a coherent framework. This theoretical framework must encompass a wide range of energy and momentum scales for nuclei ranging from the deuteron to the superheavy elements. Theory strives to describe the structure and dynamics of these often-disparate systems and to apply the understanding thus achieved to unravel some of the mysteries of the universe....



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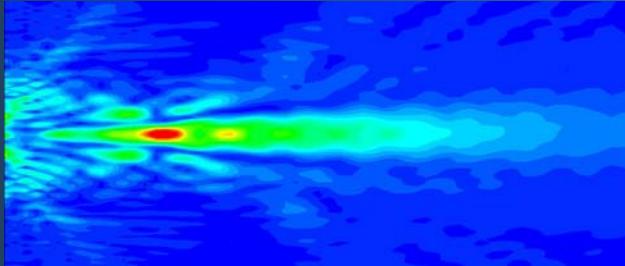
PHYSICS AND
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*CURRENT PROJECTS IN SUBATOMIC PHYSICS:
theory of nucleon structure;
symmetry tests of strong & weak interactions;
nuclear astrophysics,
precise atomic mass determinations;
ion & atom trapping*

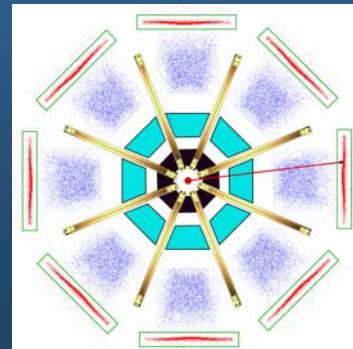
STRATEGIC LINKS:

*TRIUMF, Argonne & Los Alamos National Labs,
Jefferson Lab, Max Planck Inst.*

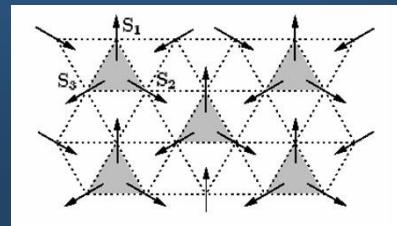


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